From Interactivity Matrix to Popularity Metrics

Supriya, Kedar, Kartheek, Ashish, Vidhyaa

April 19, 2010

Objective

- INPUTS: Interactivity matrix W, where $W(i, j)$ -
  - Number of encounters between node *i* and node *j*
  - Duration of encounters between node *i* and node *j*
  - A weighted combination
- OBJECTIVES: Develop methods to compute “Popularity Scores" for each node

Outline

Outline

Concept of distance

- Define the distance matrix $\mathcal{D}$ as

$$\mathcal{D}(i,j) = \begin{cases} 0 & \text{if } i = j, \\ \frac{1}{W(i,j)} & \text{if } i \neq j, W(i,j) \neq 0, \\ \infty & \text{if } i \neq j, W(i,j) = 0. \end{cases}$$

- Intuition: The more you meet, the closer you are

Concept of Distance

-
  - $\mathcal{D} = \frac{1}{enc}$
  - More frequently you meet, the closer you are
-
  - $\mathcal{D} = \frac{1}{dur}$
  - Longer you meet, the closer you are
-
  - $\mathcal{D} = \frac{1}{\alpha \times enc + \beta \times dur}$
  - Frequency and duration are both important!!!

Floyd-Warshall Algorithm

- All pairs shortest paths
  - FLOYD-WARSHALL(D)

```
n = D.rows
S = D
for k = 1 to n do
  for i = 1 to n do
    for j = 1 to n do
      S(i, j) = min(S(i, j), S(i, k) + S(k, j))
      if min = S(i, k) + S(k, j) then
        Π(i, j) = Π(k, j)
      end if
    end for
  end for
end for
return S
```

Popularity Scores

Useful information:
For each node *i*, we calculate

- $SP(i)$ = Sum of shortest distances from node *i* to all other nodes = $\sum_{j=1}^{n} S(i,j)$
- $TR(i)$ = Number of shortest paths passing through node *i*

Outline

Concept of Interaction Probability

- Let $c = sum(W)$
- Define the Interaction Probability matrix $\mathcal{I}_{prob}$ as

$$\mathcal{I}_{prob}(i,j) = \frac{W(i,j)}{c}$$

Concept of Interaction Probability

- If $W = enc$, then $\mathcal{I}_{prob}(i, j)$ is the probability that a randomly chosen encounter is between node i and node j
- If $W = dur$, then $\mathcal{I}_{prob}(i, j)$ is the proportion of total communication time utilized by node pair i and j
- If $W = \alpha \times enc + \beta \times dur$, then $\mathcal{I}_{prob}$ is a weighted combination of the above two factors

Modified Floyd-Warshall Algorithm

- All pairs shortest paths
  - $FLOYD_WARSHALL_PROB(I_{prob})$

$n = I_{prob}.rows$
$M_{prob} = I_{prob}$
for $k = 1$ to n **do**
  for $i = 1$ to n **do**
    for $j = 1$ to n **do**
      $M_{prob}(i,j) = \max(M_{prob}(i,j), M_{prob}(i,k) \times M_{prob}(k,j))$
      if $max = M_{prob}(i,k) \times M_{prob}(k,j)$ **then**
        $\Pi(i,j) = \Pi(k,j)$
      end if
    end for
  end for
end for
return M_{prob}

Popularity Scores

Useful information:
For each node i, we calculate

- $MP_{prob}(i)$ = Sum of maximum probability paths from node i to all other nodes = $\sum_{j=1}^{n} M_{prob}(i,j)$
- $TR_{prob}(i)$ = Number of maximum probability paths passing through node i

Outline

Outline